Gravitational Wave Emission from Galactic Radio Pulsars

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(October,1996)

We consider in this work continuous gravitational wave (GW) emission from non-axisymmetric radio pulsars. We treat in some detail the observational issues related to the known radio pulsar sample with the aim of unveiling the actual number of sources contributing to GW, which are likely to be the main contributors of GWs. It is shown that the operation of spheroidal GW detectors and full-size interferometers could detect this component of the radiation or impose useful limits on the effective oblateness of young radio pulsars.

PACS 04.40.Dg, 04.80.Nn

I. INTRODUCTION

Considerable experimental and theoretical efforts are being devoted by several groups to the physics of gravitational waves (hereafter GW). After the celebrated birth of neutrino astronomy [1], GW detection may open a new observational window related to a long-sought prediction of general relativity.

Presently, not only several resonant bars are being operated and continuously improved [2], but also the first full-scale wideband interferometers have been started and should be operational before the turn of the century [3]. The expected sensitivity of the advanced versions of these detectors in terms of the gravitational strain is of the order of $h \approx 10^{-22}$ for short-lived, impulsive bursts, and $h \approx 10^{-26}$ for periodic ("long-lived") sources, for which integration times of about 10^7 s are possible. In addition, spheroidal antennas of restricted bandwidth have been put forward again to complement the former, specially in the kHz region where they are expected to have their best performances [4]. This is precisely a range where we expect to find most of the nearest sources, namely rotating neutron stars in the Galaxy. The first positive detection(s) of GW by any of these devices will be a profound and significant achievement in physics.

Clearly, a major concern for the above detectors is to understand and minimize the sources of noise which define the sensitivity of the antennas at a given frequency. Furthermore, the important question of identifying a standard source has been put forward [5]. Among the possible candidates, it is expected that pulsars, which are known to be abundant in the Galaxy, could rank among the most conspicuous emitters of GW. Pulsars could have a time-varying quadrupole moment (and hence radiate GW) by either having a slight asymmetry in the equatorial plane (assumed to be orthogonal to the rotation axis) or a misalignment between the symmetry and total angular momentum axes, which produces a wobble in the star motion. In the former case the GW frequency is equal to

the rotation frequency, whereas in the latter two modes are possible: one in which the GWs have the same frequency as rotation, and another in which the GWs have twice the rotation frequency (the first mode dominates by far at small wobble angles while the importance of the second increases for larger values). These mechanisms have been considered in the literature [6-14] and there seems to be a certain consensus for the likelihood of a positive detection once the antennas become operative.

Generally speaking, the calculations and discussions have been focused on the detection of *individual* sources like the Tokyo experiment, devised to detect GW from the Crab pulsar [15]. However, very recent work along these lines pointed out that the radiation from the pulsar population as a whole could be detectable and paid particular attention to the expected time modulation in full-scale interferometers [16,17]. On the other hand, omnidirectional detectors (which may well be operative on shorter time scales) would not experience (by definition) any modulation and will be designed to have a very good sensitivity for continuous radiation and integration times of the order of several months. Thus, we have in principle a physical component which is different from simple noise background. We address in this work the expected features of this signal, with particular emphasis on the difficulties created by the use of the observed distribution of the sources, which is affected by several observational biases. The goals of this first approach will be to unveil as much as possible the features of the true population contributing to this emission and to discuss what can be learned from the observations of the latter.

II. PULSAR STATISTICS

In its simplest form, the evolution of a single pulsar angular frequency may be written as

$$\omega(t) = \omega_o \left(1 + \frac{t}{\tau_m} \right)^{-m/2} \tag{1}$$

where ω_o is the frequency at birth, τ_m is the characteristic e-folding time for the pulsar deceleration and m is the index of the torque power-law. From this equation, the so-called braking index is defined as $n = \ddot{\omega}\omega/\dot{\omega}^2 =$ 1+2/m. As is well known, the case of pure dipolar magnetic braking corresponds to m = 1 and hence n=3. On the other hand, the four presently determined braking indexes differ from the canonical value (2.51 for PSR0531 + 21, 2.24 for PSR0540 - 69 and 2.837 for PSR1509 - 58 where the errors affect the last significative digit, see [18] and references therein); most notably the recently announced value 1.4 ± 0.2 for the Vela pulsar by Lyne et al. [19]. The true reasons for these discrepancies are not clear as yet, but they may reflect substantial non-dipolar components of the magnetic field **B** [20], time-varying physical parameters like the amplitude of B [21], the moment of inertia I [19] or the angle between the magnetic and rotation axes [22,23]. For the moment, we shall leave m unspecified so as to allow realistic noncanonical braking scenarios as well. The braking will be determined as a byproduct of the analysis of the pulsar period distribution, and therefore should be considered in a statistical sense only and not aiming to represent any particular object.

To address the expected GW signal from the pulsar population in the galactic disk, we should first express the number of *observed* pulsars per unit volume and period interval $d^2N/dP\,dV$ as a function of pulsar-related parameters and observed quantities. This number can be written as

$$\frac{d^2N}{dP\,dV} = f_B \,\nu_p(t) \,\frac{p(R,Z)}{V_d} \,\frac{dt}{dP} \tag{2}$$

where $\nu_p(t)$ is the pulsar birthrate and V_d is the disk volume. The factor f_B accounts for the beaming effect and for most of the emission models is in the range 0.1 to 0.25. The quantity p(R,Z) is the probability to find a pulsar at a galactocentric distance R and at a height Z from the galactic plane. The pulsar observations at radio frequencies, where they have been widely studied (around $400 \, MHz$), may be used as a "window" defining the "observable" volume, since we expect that detection decreases considerably for flux densities S_{400} below 10 mJy. As usual, the flux density (at 400 MHz) is defined by $S = L_p/r^2$, with L_p being the pulsar luminosity and r the pulsar distance to the observer. Since the radio sensitivity is necessarily limited by instrumental and signal analysis [24] to those objects emitting stronger than $S_{min} \simeq 10 \, mJy$ (meaning that we have an incomplete knowledge of the true sample, see for example [25]) the integration over the volume is converted into

$$\frac{dN}{dP} = \frac{\nu_p(t)}{V_d} \frac{1}{\dot{P}} \int_{-a}^{a} 2\pi dZ \int_{S_{min}}^{\infty} p(Z(S), R(S)) \frac{L_p}{2S^2} dS \quad (3)$$

To proceed we need to evaluate the integrals in eq.(3). To this purpose we have checked that the observed pulsar luminosity function L_p can be represented to a very good accuracy by

$$L_p = A \dot{P}^{\alpha} \exp(-\beta P) \, mJy \, kpc^2 \tag{4}$$

where $A=6.4\times 10^7$, $\alpha=0.37$ and $\beta=0.982\,s^{-1}$ are the adjusted parameters.

Let us, for the moment, assume an uniform space distribution through the disk. Therefore using eqs.(2),(3) and (4) we obtain after integrating

$$\frac{dN}{dP} = f_B \nu_p(t) \left(\frac{A}{S_{min} R_d^2}\right) \left(\frac{2\tau_m}{m P_o^{2/m}}\right)^{(1-\alpha)} \times \left(\frac{exp(-\beta P)}{P^{(1-2/m)(1-\alpha)}}\right)$$
(5)

To extract the value of the unknown parameters, we proceed to plot $\log dN/dP \exp(\beta P)$ as a function of $\log P$ for the actual observed distribution (see, for example, [26,27]). This yields

$$log\left(\frac{dN}{dP}exp(\beta P)\right) = 3.00 + 0.41 log P \tag{6}$$

with a correlation coefficient of 0.904 . Therefore, by comparing eqs.(5) and (6) we get m=1.212, and thus a (statistical) braking index n=2.6, not far from the mean of value the directly observed braking indexes [18]. In order to estimate the pulsar birth rate [26], we first note that the constant in eq.(6) is defined by the relation

$$f_B \nu_p(t) \left(\frac{A}{S_{min} R_d^2} \right) \left(\frac{2 \tau_m}{m P_o^{2/m}} \right)^{(1-\alpha)} = 10^3$$
 (7)

From eq.(6) one obtains

$$\dot{P} P^{(\frac{2}{m}-1)} = \left(\frac{m P_o^{2/m}}{2\tau_m}\right) \tag{8}$$

Performing the average of the above equation, using the data from reference [27] yields

$$\left(\frac{m P_o^{2/m}}{2 \tau_m}\right) = 9.9 \times 10^{-15} \tag{9}$$

and inserting this value in eq.(7), one gets $f_B \nu_p = 10^{-3} yr^{-1}$, which corresponds to a pulsar birth rate of once every 100 to 200 yr according to the adopted value for the beaming factor. These results are in good agreement with the calculations performed by Lorimer et al. [28]. On the other hand, assuming an average initial period of pulsars of 10 ms, we conclude from eq.(9) that the average braking time is $\tau_m \approx 1022 \ yr$ and that their average lifetime is about 42.3 Myr.

Now we have expressed the observed number of pulsars as in eq.(5), we can state that the actual number is simply

dN/dP times f_B^{-1} , or going back to eq.(2) and changing , for convenience to the variable ω

$$\frac{d^2N}{d\omega\,dV} = \lambda \, p(Z,R) \, \omega^{-(1+2/m)} \tag{10}$$

where we have introduced $\lambda=(2\,\pi)^{2/m}\,\frac{\nu_p}{V_d}\,\big(\frac{2\tau_m}{mP_o^{2/m}}\big)$. With these results we turn now to the issue of calculating the GW emission expected in the detectors.

III. EXPECTED AMPLITUDE OF THE GW EMISSION

A population of pulsars having a disk distribution, and for which a slight equatorial asymmetry in each object is expected to be present, produce a non-vanishing value of the gravitational strain h^2 , whose average value is given by

$$< h^2 > = \int dV \int d\Omega \, h_*^2 \left(\frac{d^2 N}{d\omega \, dV} \right)$$
 (11)

where $h_*^2 = \frac{8}{15} \left(\frac{G}{c^4}\right)^2 \bar{\varepsilon}^2 I_{zz} \frac{\Omega^4}{r^2}$ is the angle-averaged contributed amplitude from each source [7,9] adequate for omnidirectional antennas, (see [16,17] for a discussion of the interferometer case where the orientation is important), I_{zz} is the principal moment of inertia about the z axis (assumed to be the same for all pulsars) and $\bar{\varepsilon} = (a-b)/(ab)^{1/2}$ is a mean over the individual ellipticities in the equatorial plane of the stars (a, b) being the radii along the x and y axes respectively). $\Omega = 2\pi\nu$ is the GW frequency and is equal to twice the pulsar rotation frequency ω and r is the distance to the pulsar. Note that this emission is assumed to arise from non-axisymmetric bodies and *not* from precessing ones, which have been addressed in de Araújo et al. (1994) [14] and which are likely to appear as impulsive sources (in spite of several remarks along the years some confusion remains in the literature about these cases).

Inserting eq.(10) into eq.(11) and integrating over the volume and bandwidth $\Delta\Omega$ of a resonant detector we get

$$< h^2>^{1/2} \, = 3.27 \, \times \, 10^{-24} \, \Omega_c^{0.675} \, \Delta \Omega^{1/2} \, \bar{\varepsilon} \eqno(12)$$

where Ω_c is the central frequency of the antenna. In order to obtain the numerical factor, we have adopted a disk radius of 12 kpc and that the nearest pulsar is about 500 pc. Increasing or decreasing the latter value by a factor of two produces a decrease or an increase by 20% respectively in the coefficient of eq.(12).

The case of broad-band interferometers is a bit more complicated since the signal does depend on the relative orientation and thus the signal will be modulated (see [16,17]. Although we have not attempted a detailed calculation of this modulation, our model is nevertheless useful to give an estimate of the emission strength modulo

factors of O(1). In this case we obtain, after integrating eq.(11) over the range 10-1000 Hz,

$$< h^2 > ^{1/2} = 3.3 \times 10^{-24} \Omega_{max}^{1.175} \bar{\varepsilon}$$
 (13)

where Ω_{max} is the upper frequency bound of the antenna.

IV. DISCUSSION

From the results of the former sections we can outline an optimal strategy to search for this signal. Let us consider first the case of a pair of resonant spheroidal antennae [29]. We assume a design capable of covering the frequency range 800-1000 Hz and a strain noise $\sqrt{S_h} 10^{-23} Hz^{-1/2}$, which seems reasonable for the near future. In fact, it would be possible to go as low as 600 Hz for the central frequency if the spheroidal detector can be made of niobium instead of a cheaper material like Al alloys. Such a reduction is important when the whole set of possible sources is considered and is under study by some experimental groups [29].

For each of these omnidirectional detectors, the positive identification of this GW radiation at, say, $4\,\sigma$ level requires a minimum detectable amplitude

$$h_c \simeq 4 \left(\frac{S_h}{\tau}\right)^{1/2} \tag{14}$$

where we shall assume that the strain noise $\sqrt{S_h}$ is dominated by thermomechanical forces [4,13] and τ is the integration time $\sim 10^7 \, s$. Imposing reasonable technological improvements for a pair of Al alloy " 4^{th} " generation antenna, the minimum detectable amplitude is $h_c \geq 10^{-26}$. Comparing eq.(12) and eq.(14) we deduce that the emission from the ensamble would be detectable by the array if the (mean) characteristic ellipticity of pulsars satisfies

$$\bar{\varepsilon} \ge 3.6 \times 10^{-7} \tag{15}$$

That is, the array would be sensitive to the pulsar population rotating faster than $P \simeq 2.4 \, ms$. These figure change to $\varepsilon \geq 4.7 \times 10^{-7}$ and $P \, simeq \, 3.6 \, ms$ for a niobium array with a central frequency of 600 Hz, which gives an idea of the range to be explored by these devices.

Since we have always referred to the radio pulsar population, for consistency of the picture we shall demand that $\dot{E}_{gw} < I_{zz}\omega\dot{\omega}$, where the right term represents the energy rate being extracted from the rotating neutron star. Using the well-known expression for \dot{E}_{gw} derived by Ferrari and Ruffini [6]

$$\dot{E}_{gw} = \frac{32}{5} \frac{G}{c^5} I_{zz}^2 \varepsilon^2 \omega^6 \tag{16}$$

we get that the GW emission is not dominant provided that [13]

$$\varepsilon < 1.9 \times 10^5 \left(\dot{P}P^3\right)^{1/2}$$
 (17)

For those target pulsars contributing to the pair of spheroidal antennae we get from eqs.(1) and (8), an average initial deceleration rate is $\dot{P} \approx 5 \times 10^{-13}$, and thus their average equatorial deformation must satisfy $\bar{\varepsilon} < 1.5 \times 10^{-5}$ in order to brake by electromagnetic emission. This upper limit is a factor of 40 higher then the lower limit implied by eq.(15) an)d, in a sense, the window to be probed by the detectors. It is interesting to note that the window does not shrink (in fact becomes slightly larger) if one considers the lower frequency niobium array. Although we shall not be concerned with the precise origin of the ellipticity, we note that if the crust of neutron stars is solid, then its shape may not necessarily be axisymmetric under the effect of rotation, as it would be the case for a fluid. More recent studies suggest strongly that most of the neutron star is in a liquid phase [30], and in this case other mechanisms must be invoked to produce triaxiality. Several other mechanisms, however, may induce non-axisymmetric deformations above the detectability bound of eq.(15) as discussed in refs.[31-34] (in particular, the presence of stochastic magnetic fields in the mantle of the neutron star or the existence of a type II superconductor [35]).

The possible existence of a subpopulation of silent radio pulsars whose spin-down is driven by GW [13] is also interesting and would add to h, although we have not based any of our predictions on them. We also remark that, given the uncertainties governing our knowledge of the young pulsar sample, it is not guaranteed that the initial periods of the newborn pulsars are as short as $2.4 \, ms$ (or 3.6 ms in the case of a niobium array). In fact, a direct backward extrapolation of the present Crab period gives an initial value of 4-5 times of that, or about $10 \, ms$. If this is the generic case it is unlikely that any omnidirectional array could be constructed to detect these sources, although interferometers will certainly do due to their braod-band coverage and maximum sensitivity at lower frequencies. A complicated temporal modulation pattern will arise in the latter, which may nevertheless help to extract this signature (see [16]). An estimate of the signal strength can be obtained from our model by comparing eq.(13) with the maximum sensitivity region of the advanced interferometers, giving the detectability condition

$$\bar{\varepsilon} \ge 7 \times 10^{-7} \tag{18}$$

to be compared with the upper bound $\bar{\varepsilon} \leq 8.5 \times 10^{-5}$. The region to be explored is now about two orders-of-magnitude in the $\bar{\varepsilon}$ parameter.

To understand what do the detectability conditions really mean we have collected in Table 1 the relevant numbers of five "top candidates" according to the standard procedure of assuming that all the observed \dot{P} is due to GW emission. This bold assumption yields a maximum ellipticity ε_m and therefore the maximum amplitude h_m to be expected for that individual sources. Table

1 stresses the importance of exploring the hidden sources we have been referring to: while the largest h_m belong to young pulsars like Crab and Vela, the required ε_m seem to be rather extreme (a hint for the incorrectness of the "all GW" assumption); on the other hand ms pulsars like PSR1957+20 can not be strong GW emitters because their total energy loses are known to be small. Therefore the ideal emitters would be rapidly rotating, high \dot{P} young pulsars which may populate the upper corner of the $\dot{P}-P$ diagram, having the additional advantage of possibly falling in both the spheroidal and interferometric detector ranges. It can be said that the search for an optimal experimental situation has forced us to address the statistical expectations for these yet undiscovered sources.

Source	$arepsilon_m$	h_m	ν_{GW} [Hz]
Vela	1.8×10^{-3}	1.9×10^{-24}	22.5
Crab	7.5×10^{-4}	1.5×10^{-24}	60.6
Geminga	2.3×10^{-3}	1.2×10^{-24}	8.4
PSR1509-68	1.4×10^{-2}	5.8×10^{-25}	13.2
PSR1957+20	1.6×10^{-9}	1.7×10^{-27}	1244

TABLE I. Upper limits to the GW emission from individual sources (see text)

It is to be noted that, according to their small asymmetry as deduced from their observed features [13], the millisecond pulsar subpopulation would not contribute to the ensamble radiation. Consistently, this subpopulation has not been included when considering the statistical features exposed in Sec.2. This does not mean, however, that ms pulsars are not interesting for GW searches (see, for example, [13,14]), but rather that our calculations do not depend on them. We believe that the accumulated evidence for a distinct evolutionary path for these objects is enough for a separate treatment of their emission.

As a final remark it is important to stress that, in spite of the statistical treatment given in this work, $\langle h^2 \rangle^{1/2}$ will be dominated by the youngest pulsars in the Galaxy because of the rather strong dependence of the gravitational emission on ω . For example, from our statistical analysis we found that the total number of pulsars with $P \leq 4s$ is around 10^5 , but only ~ 25 objects would be responsible for 70% of the total gravitational radiation in the interferometers. We would also predict in our model that only ~ 3 of them would be detected at radio frequencies $(400 \ Mhz)$ in the period range 10-30 ms. The true situation may be more optimistic than we think. According to Camilo [36], a clue about the young pulsar population may be hidden in the $\dot{P}-P$ diagram. His argument is precisely that evolution along a $\mathbf{B} = constant$ line may require the presence of many young pulsars with $\dot{P} \geq 10^{-14} s/s$ and $P \leq 50 \, ms$ which remain undiscovered because of selection effects and are missed in the existing searches. This independent evidence reinforces the motivation of our statistical approaches and calls for future refinements. Detection of this GW radiation would be an important check for the first GW observatories, and even a non-detection would be useful to set constraints on the shape an on the statistics of young galactic pulsars.

J.E.H. wishes to acknowledge the hospitality and financial support from Department Fresnel, O.C.A. during a visit which made this work possible and a Research Fellowship from the CNPq Agency (Brazil).

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